Double-bounded pay factors for determining payment of in-place concrete compressive strength

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Introduction

Under a double-bounded pay-factor system, in-place concrete whose compressive strength is exceedingly higher or lower than a design mean is paid at a lower rate than material closer to the target. Pay factors determine the amount of payment reward or penalty that is applied to an in-place lot of 28-day concrete compressive strengths (CCS). These pay factors vary around the design mean, enticing the placement of concrete closer to that target.

Existing guidance for the use of a double-bounded pay factor system for the placement of concrete is inadequate if the design distribution is not a Normal distribution. The existing guidance assumes that incentives and disincentives offered by the pay factors are symmetrical around the design mean. It is unlikely that states will want to have symmetrical incentives around the design mean, since strengths exceeding the mean are not nearly as problematic as strengths falling short of the mean. The team also found that the industry standard for the determination of payment from a sample lot of CCS, the percent within limits (PWL), lacks adequate resolution for an asymmetric double-bounded system with a design mean. Any set of CCS samples that fall securely within the bounds can result in a PWL at or near 100%, making it difficult to reward lots that are within the acceptance range *and* close to the design mean. The following tasks were undertaken for this project:

1. **Develop a new methodological approach for calculating payment from a distribution of 28-day CCS that is not dependent on the distribution type**
2. **Demonstrate the implementation of the new approach for 3 forecast scenarios**

New Methodological Approach

For the first task, the team found the PWL to be inadequate for the application of a double-bounded pay factor system with an off-center design mean and instead developed the *Percent Within Distribution* (PWD), which provides more resolution within the acceptance range for CCS. The PWD is represented by the fraction of the distribution suggested by sample lot that falls within the design distribution, as shown in Figure 1. The algorithm takes in a design mean, standard deviation, and an upper and lower acceptance boundary. It uses this data to calculate a PWD for any size lot of samples using a Bayesian approach to re-estimating the distribution parameters for each sample, then calculating the PWD by comparing the final distribution with the design distribution after all samples have been passed through the Bayes algorithm. The Bayes approach is ideal for this application because the contractor has knowledge of the design distribution, so samples are safely assumed to be new realizations of that distribution, with variations in the parameters.

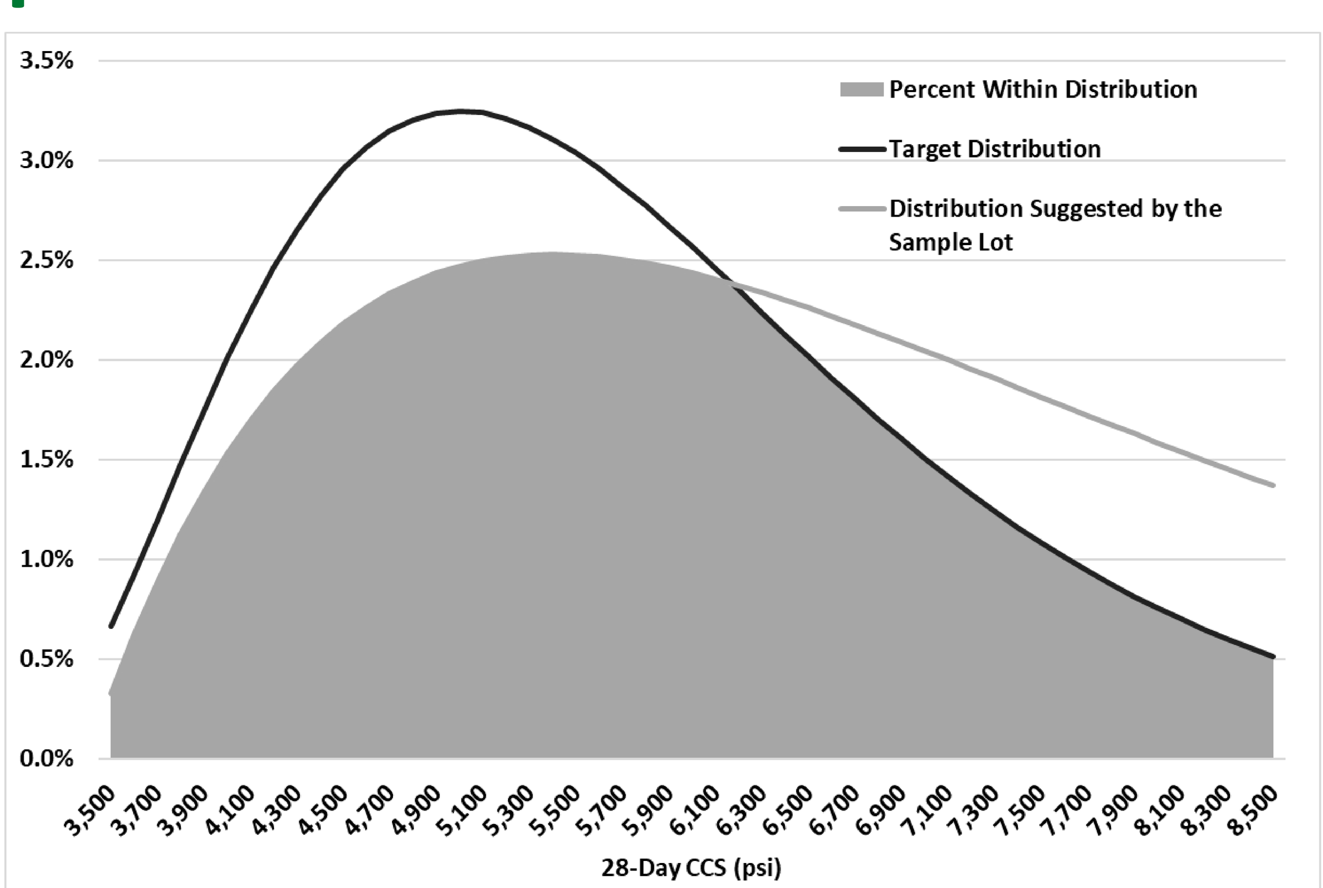

Figure 1. Percent Within Distribution (PWD) calculated from a Design Distribution and a Distribution Suggested by the Sample Lot

Forecast Scenarios

For the second task, the research team simulated 900 randomized industry responses to a double-bounded design distribution, with the design mean and design standard deviation shown in Table 1. Each of three response types was simulated, indicating the extent to which the industry responds to the pay factors – no response, partial response, or dramatic response.

Table 1. Simulation Specifications and Parameters

Design Parameters	Mean	5,500
	Std Dev	500
Specification Limits	Upper	8,000
	Lower	4,000
Simulation Parameters	Samples	6
	Simulations	900

First, a translation of PWDs to pay factors was created by simulating "perfect" lots directly from the design distribution. These simulations indicated that meeting the design distribution resulted in an average PWD of about 92%, with a standard deviation of about 4%. Therefore, PWDs as low as 88% should still be paid at the maximum pay factor. This finding created a "calibration point" for pay factors against PWDs. Simulated industry responses were:

1. **Mild response – no shift in the distribution of 28-day CCS is observed but high-strength samples are avoided; random samples were selected between the upper and lower specification limits.**
2. **Partial response – partial shift in the mean of the distribution of 28-day CCS is observed; random samples were selected between 7,600 psi and the lower specification limit.**
3. **Dramatic response – shift in the mean or sd of 28-day CCS is observed; random samples were selected from the design distribution, except with the mean at 5,700 and sd at 500, or with the mean at 5,500 and the sd at 750.**

The results, shown in Table 2, are sets of pay factors for each scenario that balance Agency and industry risk by yielding an overall cumulative reward payment of 1.03. PWDs higher than 96% are rare, and PWDs lower than 32% will often include samples outside of the specification limits.

Table 2. Simulation Results

	Pay Factors for Scenario		
PWD	1	2	3
96%	1.07	1.06	1.04
92%	1.07	1.06	1.04
88%	1.07	1.06	1.04
84%	1.06	1.05	1.03
80%	1.05	1.04	1.02
76%	1.04	1.03	1.01
72%	1.03	1.02	1.00
68%	1.02	1.01	0.99
64%	1.01	1.00	0.98
60%	1.00	0.99	0.97
56%	0.99	0.98	0.96
52%	0.98	0.97	0.95
48%	0.97	0.96	0.94
44%	0.96	0.95	0.93
40%	0.95	0.94	0.92
36%	0.94	0.93	0.91
32%	0.93	0.92	0.90

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